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SIMILITUDES BETWEEN THE METHODS USED TO STUDY ENVELOPING SURFACES
II. SURFACES RECIPROCAL RELIVED TO HELICOIDS

Prof.dr.ing. Nicolae OANCEA, Conf.dr.ing. Gabriel FRUMUŞANU,
Universitatea "Dunărea de Jos" din Galaţi

ABSTRACT

Analitical demonstrations of the similitude between the enveloping condition forms, as they result from using the different methos to study the enveloping surfaces of the cylindric helical surfaces, by constant pitch, are exposed in this paper. It is proved the equivalence between the methodes: Nikolaev, "Minumum Distance", "Substituting Circles", "Trajectories" and Gohman's Theoreme. It is also proven that, for all these methods,the fact that the condition to determine the characteristic curve from the contact between a cylindric helical surface, by constant pitch and a revolution one can be brang to a unique analitical expression, and through this, the validity of these methods it is highlited.

1. The Revolution Surface Associated to a Helical Surface

Among the problems connected to the field of disc tools or end mill cutters profiling, an important one is to find a revolution surface as reciprocal enveloped to a cylindric helical surface (with constant pitch).

The solution of this problem can be determined by using a number of different algorithms: Gohman's Method [1,2], Nikolaev's Condition [3], Minimum Distance Method [2,6], Substituting Circles Method [2,7], Trajectories Method [2,8].

By using specific ways of presentation, the mentioned methods seem to be different; however, a detailed analysis shows a unique solution, no matter what method being used.

It must be reminded that, when solving the problem of the contact between helical and revolution surfaces, an important step is to find the characteristic curve (tangent curve) of the associated surfaces couple; in the case of each upper mentioned method, the condition to find the characteristic curve has a specific aspect.

Starting from here, we can prove that the condition to find the characteristic curve, by using any of the methods, is also unique.

2. GOHMAN's Method [1,2]

If the cylindric helical surface - Σ, having constant pitch, $\vec{V}$ axis and p as helical characteristic value is known as:

$$\Sigma \begin{vmatrix} X = X(u,v); \\ Y = Y(u,v); \\ Z = Z(u,v), \end{vmatrix} \qquad (1)$$

the purpose is to find a revolution surface -S- the first peripheral surface of the future tool (disc or end mill cutter) of $\vec{A}$ axis, disjunct to the helicoid $\vec{V}$ axis.

When generating the helical surface, the revolution surface is executing a rotation motion around its axis; in the same time, the

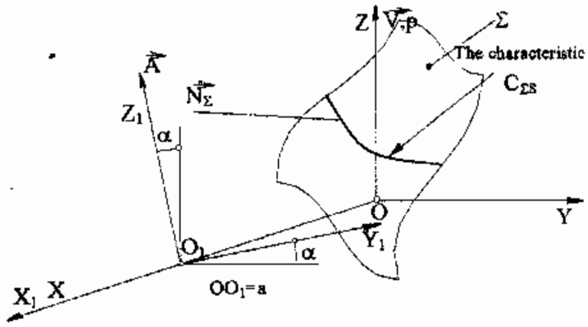

Fig.1 GOHMAN's Condition

surface Σ has a helical motion around $\vec{V}$ axis and by p helical characteristic value, self-generating motion in the case of the surface Σ.

As it follows, the characteristic curve of Σ being not related to the motion component that leads to the surface selfgenerating, the rotation motion around the $\vec{A}$ axis results as unique motion to define the characteristic curve.

So, by definig Σ reffering to the $X_1Y_1Z_1$ system and by using the co-ordinates change

$$\begin{Vmatrix} X_1 \\ Y_1 \\ Z_1 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{Vmatrix} \cdot \begin{Vmatrix} X(u,v)-a \\ Y(u,v) \\ Z(u,v) \end{Vmatrix}, \qquad (2)$$

or

$$\Sigma \quad \begin{cases} X_1 = X(u,v) - a; \\ Y_1 = Y(u,v)\cdot\cos\alpha + Z(u,v)\cdot\sin\alpha; \\ Z_1 = -Y(u,v)\cdot\sin\alpha + Z(u,v)\cdot\cos\alpha, \end{cases} \qquad (3)$$

during the motion

$$X_1 = \omega_3^T(\varphi)\cdot \begin{Vmatrix} X(u,v)-a \\ Y(u,v)\cdot\cos\alpha + Z(u,v)\cdot\sin\alpha \\ -Y(u,v)\cdot\sin\alpha + Z(u,v)\cdot\cos\alpha \end{Vmatrix}, \qquad (4)$$

the family of surfaces Σ is generated:

$$(\Sigma)_{\varphi 1} \begin{cases} X_1 = [X(u,v)-a]\cdot\cos\varphi - [Y(u,v)\cdot \\ \quad \cdot\cos\alpha + Z(u,v)\cdot\sin\alpha]\cdot\sin\varphi; \\ Y_1 = [X(u,v)-a]\cdot\sin\varphi + [Y(u,v)\cdot \\ \quad \cdot\cos\alpha + Z(u,v)\cdot\sin\alpha]\cdot\cos\varphi; \\ Z_1 = -Y(u,v)\cdot\sin\alpha + Z(u,v)\cdot\cos\alpha. \end{cases} \qquad (5)$$

The enveloping condition (to find the characteristic curve) will be

$$\vec{N}_{\Sigma_1}\cdot\vec{R}_\varphi = 0, \qquad (6)$$

where : $\vec{N}_{\Sigma_1}$ is the normal to the surface Σ, given in the $X_1Y_1Z_1$ reference system;

$\vec{R}_\varphi = \dfrac{dX_1}{d\varphi}$ is the velocity vector in the motion of the surface Σ respect to the $\vec{A}$ axis,

$$R_\varphi = \begin{Vmatrix} -\sin\varphi & -\cos\varphi & 0 \\ \cos\varphi & -\sin\varphi & 0 \\ 0 & 0 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} X(u,v)-a \\ Y(u,v)\cdot\cos\alpha + Z(u,v)\cdot\sin\alpha \\ -Y(u,v)\cdot\sin\alpha + Z(u,v)\cdot\cos\alpha \end{Vmatrix} \qquad (7)$$

By denominating the components of the normal to the Σ surface (1) as N_x, N_y and N_z, then, refering to $X_1Y_1Z_1$ system, their form is

$$\begin{Vmatrix} N_{X_1} \\ N_{Y_1} \\ N_{Z_1} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{Vmatrix} \cdot \begin{Vmatrix} N_X \\ N_Y \\ N_Z \end{Vmatrix}. \qquad (8)$$

As known, the characteristic curve of the surfaces Σ and S can be found by giving a constant value to φ, φ=0 for example. As consequence, the $\vec{R}_\varphi$ vector will be expressed, starting from (6) as

$$\begin{Vmatrix} R_{\varphi X_1} \\ R_{\varphi Y_1} \\ R_{\varphi Z_1} \end{Vmatrix} = \begin{Vmatrix} -[Y(u,v)\cdot\cos\alpha + Z(u,v)\cdot\sin\alpha] \\ X(u,v)-a \\ 0 \end{Vmatrix}. \qquad (9)$$

Thus, having in view (6), (7) and (8), the condition to find the characteristic curve becomes

$$\begin{aligned} &-[Y(u,v)\cdot\cos\alpha + Z(u,v)\cdot\sin\alpha]\cdot N_X \\ &+[X(u,v)-a]\cdot(N_Y\cdot\cos\alpha + N_Z\cdot\sin\alpha) = 0 \end{aligned}. \qquad (10)$$

3. NIKOLAEV's Condition [2,3]

Nikolaev's Condition, based on decomposing the helical motion on disjunct axis rotation motions, Fig.2, leads to the conclusion that, the characteristic curve, common to both conjugated surfaces, helical - Σ and revolution - S, is identical with the projection of $\vec{A}$ axis on the helical surface, Σ.

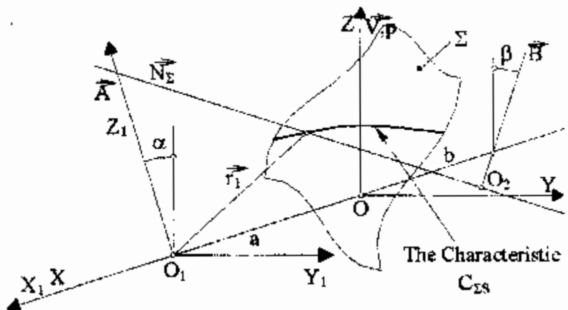

Fig.2. NIKOLAEV's Condition

Otherwise, the condition to find the characteristic curve can be reduced to the a coplanarity condition imposed to $\vec{N}_\Sigma$, $\vec{A}$ and $\vec{r}_1$, where:

$\vec{N}_\Sigma$ is the normal on the surface Σ;

$\vec{A}$ - the axis of the tool having a revolution first peripheral surface;

$\vec{r}_1$ - vector between two points placed on $\vec{A}$ şi $\vec{N}_\Sigma$.

Thus, refering to XYZ system, the vectors can be defined as:

$$\vec{N}_\Sigma = N_X\cdot\vec{i} + N_Y\cdot\vec{j} + N_Z\cdot\vec{k}; \qquad (11)$$

$$\vec{A} = -\sin\alpha\cdot\vec{j} + \cos\alpha\cdot\vec{k}; \qquad (12)$$

$$\vec{r}_1 = [X(u,v)-a]\cdot\vec{i} + Y(u,v)\cdot\vec{j} + Z(u,v)\cdot\vec{k}, \qquad (13)$$

as position vector of the current point Σ, (1), refered to O_1.

The coplanarity condition applied to the three vectors is given by the mixt product

$$(\vec{A}, \vec{N}_\Sigma, \vec{r}_1) = 0, \qquad (14)$$

developed as

$$\begin{vmatrix} N_X & N_Y & N_Z \\ 0 & -\sin\alpha & \cos\alpha \\ X(u,v)-a & Y(u,v) & Z(u,v) \end{vmatrix} = 0, \quad (15)$$

or

$$[X(u,v)-a]\cdot[N_Y\cdot\cos\alpha + N_Z\cdot\sin\alpha] - \\ -N_X\cdot[Z(u,v)\cdot\sin\alpha - Y(u,v)\cdot\cos\alpha] = 0 \quad (16)$$

It is obvious that (see also (10)) the two conditions to find the characteristic curve $C_{\Sigma S}$, GOHMAN and NIKOLAEV, are identical.

4.The "Minimum Distance"Method [2,4,5,6]

It is already known the solution Nikolaev to find the enveloping condition (to determine the characteristic curve) when generating helical surfaces by using revolution first peripheral surface tools,

$$(\vec{A}, \vec{N}_\Sigma, \vec{r}_1) = 0, \quad (17)$$

see also Fig.3. condition expressing coplanarity of $\vec{N}_\Sigma$ and $\vec{A}$ vectors.

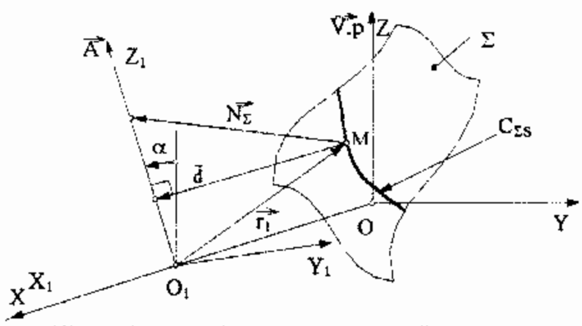

Fig.3. The "minimum distance" method

As consequence, the plain given by $\vec{N}_\Sigma$ and $\vec{A}$ can be defined having in view also the vector $\vec{d}$ - the distance between the current point on the characteristic curve and $\vec{A}$ axis,

$$\vec{d} = \lambda\cdot\vec{N}_\Sigma - \beta\cdot\vec{A}, \quad (18)$$

where λ and β are scalars.

In this way, the mixt product

$$(\vec{N}_\Sigma \times \vec{A})\cdot\vec{r}_1 = 0 \quad (19)$$

will be equivalent to

$$\{\vec{N}_\Sigma \times [\lambda\cdot\vec{N}_\Sigma - \vec{d}]\}\cdot\vec{r}_1 = 0 \quad (20)$$

or

$$[(\vec{N}_\Sigma \times \vec{d})\cdot\vec{r}_1] = 0. \quad (21)$$

Trully, by defining the plain tangent to the surface Σ in the current point of the characteristic curve, the plain admiting $\vec{N}_\Sigma$ direction as normal, all the curves placed in the surface Σ and including the poin have the property of being tangent to the plain.

Thus, also in the plain determined by $\vec{N}_\Sigma$ and $\vec{A}$ directions (the plain containing the reciprocal enveloping to Σ surface axis), the curve placed on the surface Σ is tangent to the plain admiting $\vec{N}_\Sigma$ as normal.

Obviously, in this case the point M is at the minimum distance respect to the $\vec{A}$ axis. We can also specify that the plain determined by $\vec{A}$ şi $\vec{N}_\Sigma$ is normal on the plain tangent to Σ in M.

In the same time, in any other plain including M and having $\vec{A}$ as normal, the intersection curve between itself and the surface Σ has the point M - from the tangent plain - placed at minimum distance respect the $\vec{A}$ axis.

By considering the equations of the helical surface reffered to the system $X_1Y_1Z_1$, solidary with the revolution tool axis like:

$$\Sigma \begin{vmatrix} X_1 = X_1(u,v); \\ Y_1 = Y_1(u,v); \\ Z_1 = Z_1(u,v), \end{vmatrix} \quad (22)$$

in the plain including M and having $\vec{A}$ as normal,

$$Z_1(u,v) = H, \text{ equivalent to } v = v(u). \quad (23)$$

the distance between $\vec{A}$ axis to surface Σ can be expressed as

$$\delta = \sqrt{X_1^2(u) + Y_1^2(u)}. \quad (24)$$

So, the minimum condition of the distance δ becomes

$$X_1(u)\cdot X_1'(u) + Y_1(u)\cdot Y_1'(u) = 0, \quad (25)$$

and represents the condition to find the characteristic curve at the contact between the helical surface and the revolution one.

5. The "Substituting Circles Family" Method [2,7]

The application of the "substituting circles family" method to determine the contact between a cilyndric helical surface having constant pitch and a revolution surface leads to specific conditions to find the characteristic curve.

Trully, in the case of a revolution first peripheral surface tool (Fig.4), the $C_{\Sigma H}$ curve family of substitution circles

$$C_{\Sigma H}: X_1 = X_1(u);\ Y_1 = Y_1(u), \quad (26)$$

having their centers on X_1 axis, has the following equations:

$$(C_r)_H \begin{vmatrix} X_1 = -r_i\cdot\cos\beta_i - \lambda_i; \\ Y_1 = -r_i\cdot\sin\beta_i, \end{vmatrix} \quad (27)$$

with λ_i, r_i and β_i variable.

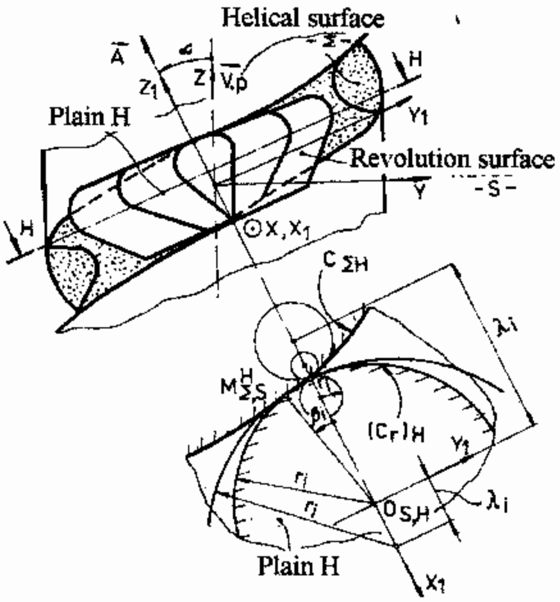

Fig.4 The "substituting circles family" method. Contact between a helical and a revolution surface

As resulting from the II[nd] theoreme [2,7], when $\lambda_i=0$, the condition that $C_{\Sigma H}$ - the intersection curve between Σ and the plain $Z_1=H$ and $(C_r)_H$ - the family of substituting circles - must have a tangence point, the following equations system results:

$$\begin{aligned} &X_1(u) = -r_i \cdot \cos\beta_i;\\ &Y_1(u) = -r_i \cdot \sin\beta_i;\\ &X_1'(u) = r_i \cdot \sin\beta_i;\\ &Y_1'(u) = -r_i \cdot \cos\beta_i, \end{aligned} \qquad (28)$$

which allows to find the condition

$$tg\beta_i = \frac{Y_1(u)}{X_1(u)} = -\frac{X_1'(u)}{Y_1'(u)} \qquad (29)$$

or

$$X_1(u) \cdot X_1'(u) + Y_1(u) \cdot Y_1(u) = 0 \ . \qquad (30)$$

The condition (30) is identical as meaning to (25), the last one being deducted when applying the "minimum distance" theoreme.

6. The "Trajectories" Method [2,8]

To solve the problem in this way, the hypothesys that the end mill cutter axis is normal to the helical surface axis (Fig.5), must be considered (it is, in fact, the usual case when using an end mill cutter).

If the cilyndric helical surface is given by the equations (1), in the plain

$$X = H_i \ (H_i \text{ variable}) \qquad (31)$$

the helical surface plain section has, in principle, the equations

$$\Sigma_H : Y = Y(u); Z = Z(u). \qquad (32)$$

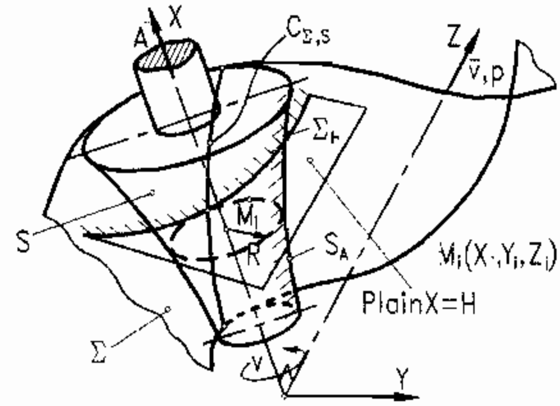

Fig.5 - The end-mill cutter

A point $M_i(H_i, Y_i, Z_i)$ from the tool first peripheral surface - S - describes a trajectory - T- during the rotation motion of this surface around the $\vec{A}$ axis, given by

$$\begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos v & -\sin v \\ 0 & \sin v & \cos v \end{Vmatrix} \cdot \begin{Vmatrix} H_i \\ Y_i \\ Z_i \end{Vmatrix}, \qquad (33)$$

with variable v, or

$$T \begin{cases} X = H_i; \\ Y = Y_i \cdot \cos v - Z_i \cdot \sin v; \\ Z = Y_i \cdot \sin v + Z_i \cdot \cos v. \end{cases} \qquad (34)$$

The point M_i is placed on the surface S - the tool first peripheral surface - if the trajectory T is tangent to the curve Σ_H.

The identification conditions of the T trajectory suppose the existance of the conditions:

$$\begin{aligned} &Y(u) = Y_i \cdot \cos v - Z_i \cdot \sin v;\\ &Z(u) = Y_i \cdot \sin v - Z_i \cos v, \end{aligned} \qquad (35)$$

representing the identity of the point M_i on the T trajectory to a point on the curve Σ_H and

$$\begin{aligned} &Y'(u) = -Y_i \cdot \sin v - Z_i \cdot \cos v;\\ &Z'(u) = Y_i \cdot \cos v - Z_i \sin v, \end{aligned} \qquad (36)$$

representing the identity of the versors giving the direction of the tangent to the curve Σ_H and those of the trajectory T (34).

From both (35) and (36) equations results the condition

$$\begin{aligned} &Y(u) \cdot Y'(u) + Z(u) \cdot Z'(u) = (Y_i \cdot \cos v - Z_i \cdot \sin v) \cdot \\ &\cdot (-Y_i \cdot \sin v - Z_i \cdot \cos v) + (Y_i \cdot \sin v - Z_i \cdot \cos v) \cdot \\ &\cdot (Y_i \cdot \cos v - Z_i \cdot \sin v) \end{aligned} \qquad (37)$$

or, after developing

$$Y(u) \cdot Y'(u) + Z(u) \cdot Z'(u) = 0 \qquad (38)$$

It is obvious that the condition (38) are similary to the conditions to find the characteristic curve through other methods.

7. Plain Generating Curves Method

The problem of generating helical surfaces by using revolution first peripheral surfaces tools can also be solved by using the "Plain Generating Curves" method.

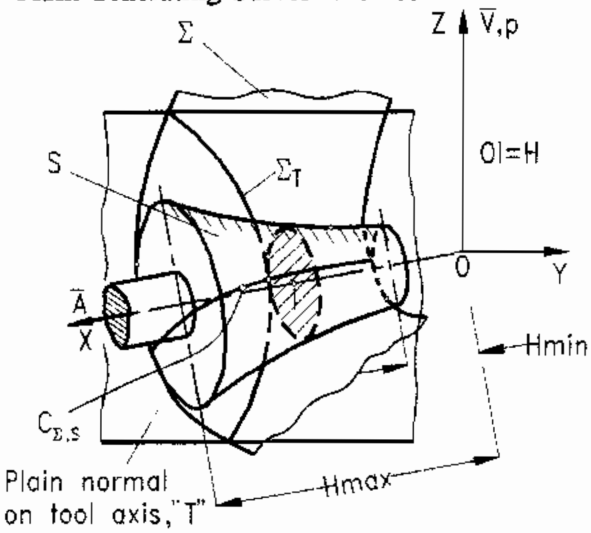

Fig.8 - "Plain Generating Curves" method applied to profile an end-mill cutter

We must observe that, in the known generating cases (with an end mill cutter, with disc-tool, with cylindric shape tools), all the time the contact between the helical surface and the first peripheral tool surface canbe examined also as a 2D problem, in the sections normal to the tool rotation axis or, in the particular case of the cylindric surface, in the plain including its generator-line (Fig.8).

In this way, the profiles Σ_T, representing the cuts of the surface Σ by the plains T (normal plains), envelop, in this plain, the tools first peripheral surfaces, allowing to find $M_{\Sigma,S}$ points, situated on the characteristic curves - the curves of tangence between the surface Σ and the tools first peripheral surface.

When generating helical surfaces with end mill cutters, the reference systems and the relative position between the tool and the generated surface should be considered like in Fig.8. Thus, XYZ is the reference system attached to the tool, the $\vec{A}$ axis being overposed to the X axis.

If the normal plain T, situated at the distance H respect to YZ plain, is considered, the intersection between this plain and the generated surface Σ is a plain curve, Σ_T.

In the case when the helical surface is given by (1), the normal section of the surface Σ when cut by the plain

$$X(u,v)=H,\ H\text{ - variable,} \qquad (39)$$

in principle determines a plain curve Σ_T,

$$\Sigma_T:\ Y=Y(u);\ Z=Z(u), \qquad (40)$$

the condition (39) being equivalent to a dependence

$$v=v(u). \qquad (41)$$

During the rotation of the curve Σ_T around the axis $\vec{A}$

$$x=\omega_1^T(\varphi_1)\cdot X, \qquad (42)$$

the family of generating curves Σ_T type is described

$$(\Sigma_T)_{\varphi_1}\left|\begin{array}{l} X=H; \\ Y=Y(u)\cdot\cos\varphi_1-Z(u)\cdot\sin\varphi_1; \\ Z=Y(u)\cdot\sin\varphi_1+Z(u)\cdot\cos\varphi_1. \end{array}\right. \qquad (43)$$

The enveloped of the Σ_h curves family represents the profile of the first peripheral tool, in the plain H.

The enveloping condition

$$\frac{Y_u'}{Y_\varphi'}=\frac{Z_u'}{Z_\varphi'}, \qquad (44)$$

having also in view (43), after calculus becomes

$$Y(u)\cdot Y'(u)+Z(u)\cdot Z'(u)=0. \qquad (45)$$

Obviously, the equation (45) is identical as form to the enveloping condition resulted from applying the "Minimum Distance Method" and equivalent to the other known metods.

By puting together the equations (43) and (44), when H takes different values, along the X axis, in such a way as the usefull part of the surface Σ to be covered ($H_{min}\le H\le H_{max}$), the characteristic curve $C_{\Sigma,S}$ on the surface Σ is determined.

8. Conclusions

Based on the specific theorems (Gohman, Willis, "minimum distance", "substituting circles family", "trajectories"), the enveloping conditions when generating surfaces associated to rolling axoids were determined.

The results prove the unicity of the enveloping condition, no matter what was the form of expressing the specific theorems.

In the same time, we must highlight the fact that these different theorems, each one with its particular enounce, even if in substance have the same meaning, by their specificity can lead to a significant calculus simpification [9].

The contact between a cylindric helical surface, with constant pitch and a revolution surface was studied in the same manner.

In this case also, althought appearently there are great differences between the specific theorems enounces, the unicity of the contact condition was proved - in fact a demonstration of the Nikolaev, "Minimum Distance", "Family of Substituting Circles", "Trajectories" theorems etc. respect to Gohman's Theoreme.

The choice between one or another of the enounced theorems depends on the followed purpose:

- a generic solution of the problem;
- a rapid and simple calculus;
- the expression of the enveloping process with more eloquence

and last but not least, on the character of the person which intends to solve the problems conected to the surfaces enveloping.

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SIMILITUDINI ALE METODELOR PENTRU STUDIUL SUPRAFEŢELOR ÎN ÎNFĂŞURARE
II. SUPRAFEŢE RECIPROC ÎNFĂŞURĂTOARE ELICOIDALE

REZUMAT

În lucrare, se prezintă, în formă analitică, demonstraţii ale similitudinii formei condiţiilor de înfăşurare, exprimate prin diferite metode de studiu a înfăşurătoarelor suprafeţelor elicoidale cilindrice şi de pas constant. Este prezentată echivalenţa metodelor: Nikolaev, "Metoda distanţei minime", "Metoda cercurilor substitutive", "Metoda traiectoriilor" faţă de teorema Gohman. Se demonstrează faptul că, pentru toate metodele enunţate, condiţiile de determinare a curbei caracteristice de contact a unei suprafeţe elicoidale cilindrice şi de pas constant cu o suprafaţă de revoluţie pot fi aduse la o exprimare analitică unică şi, prin aceasta, corectitudinea acestor metode.

SIMILITUDES PARMI LES MÉTODES POUR L'ÉTUDE DES SURFACES ENVELOPPÉES.
II. SURFACES ASSOCIÉES À DES CENTROIDES EN ROULEMENT

RÉSUMÉ

Dans ce papier sont présentées, sous forme numérique, démonstrations de la similitude de la forme des conditions d'envelopment, exprimées par différentes métodes d'étude pour les envelopées des surfaces hélicoidales cylindriques ayant pas constant. L'echivalence parmi les métodes: Nikolaev, "Métode de la distance minimale", "Métode des cercles substitutives", "Métode des trayectoires" et la Téorème Gohman est aussi présentée. On prouve que, pour toutes les métodes y antérieurement mentionnées, la condition pour déterminer la courbe caractéristique de contact d'une surface hélicoidale cylindrique ayant pas constant avec une surface de révolution peuvent avoire une expression analitique unique.